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Phenomenological Modeling of Diesel Spray with Varying Injection Profile

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ABSTRACT

Accurate and quick prediction of spray characteristics such as spray penetration is paramount for the understanding and quantitative analysis of combustion process in diesel engines, in order to perform parametric study on advanced combustion process in diesel engines, zero-dimensional diesel spray model is often used for the prediction of the spray evolution. In this study, a previous zero-dimensional diesel spray model that applied for the spray penetration prediction including the part after the end of injection (EOI) with constant injection rate was extended to the cases with varying injection rate. The effective injection velocity was introduced into the previous spray model by being defined as the ratio of the momentum flux and fuel mass flow rate over the spray tip cross-sectional area. Combining with this definition, the analysis of effective injection rate and its response time were performed during and after the EOI. After that, the fuel mass flow rate and momentum flux over the spray tip cross-sectional area were derived for varying injection rate even after EOI based on the momentum and fuel mass conservation along the spray's axis, and further the spray penetration. Finally, the developed model was validated by comparing with experimental data.

Keywords: *Diesel engine, Diesel spray, Spray model, Varying injection rate, After the end-of-injection.*

1. INTRODUCTION

Better understanding of in-cylinder processes is a prerequisite for further development and optimization of engine combustion systems, especially with regard to the increasingly stringent emission regulations. Since pollutant formation and combustion efficiency are highly dependent on mixing process of fuel and air, and fuel-air mixing is greatly influenced by spray characteristics such as penetration and cone angle. Therefore, it is of great importance to conduct researches on the prediction of spray propagation. Spray models, as effective approaches with low computational cost to achieve this target, are widely employed to perform the modeling and simulation of diesel engines combustion process.

In the past decades, there have been several models to study the spray penetration, semi-empirical or numerical.

Wakuri et al. [1] and Dent [2], found that the spray penetration distance shows a relationship of square root dependence on time. Reitz [3] and Hiroyasu [4], based on the data from a number of experiments, divided the spray propagation into two stages (before and after break-up time), got respective formula for each stage. Sazhin [5] developed a spray penetration model on the basis of two-phase theory. In addition, Naber and Siebers [6], as well as Desantes JM [7] proposed more detailed spray model according to the conservation of momentum flux and mass flow rate along the spray axis. These models have been demonstrated to be consistent with the results of experimental data and CFD computations. Nevertheless, all of these can only be applied for the cases of spray propagation in quasi-steady state during injection and without any information after the end-of- injection.

During the recent decades, the fulfilment of the increasingly demand to alleviate the environmental pollution and improve the thermal efficiency has become the most challenging task for diesel engines [8,9]. Therefore, in order to reduce fuel consumption, pollutant emissions and increase the thermal efficiency simultaneously, more and more advanced combustion strategies, such as low temperature combustion (LTC) and premixed charge compression ignition (PCCI), have been applied into modern diesel engines. Applications of these new combustion strategies contributes to longer ignition delay in modern diesel engines, so that the ignition occurs typically later than EOI, which means longer mixing time. Although the low temperature caused by the late ignition exacerbates mixture process, the combined effect on the fuel and air mixture formation tends to be positive. Obviously, the formation of the fuel-air mixture, as well as the subsequent combustion, are greatly influenced by the spray behaviour after EOI, so it is of great importance to understand and clarify the spray evolution and mixing process after EOI[10-12]. Furthermore, in order to realize advanced combustion modes, short injection duration with high injection pressure has been widely used. And the short injection duration makes the ramping-up and ramping-down processes influence more remarkably during injection, so the prediction of spray penetration cannot be considered as quasi-steady state any longer as before. Hence, a phenomenological spray model that can predict the spray evolution with varying injection profile and even after EOI is needed for parametric study on advanced diesel engines.

To analyze the spray behavior after EOI, a one-dimensional spray penetration model was proposed by Musculus and Kattke [13] based on the momentum theory and conservation of mass flow rate along the diesel jet axis for discrete control volumes. The results showed that after EOI, the decelerating state goes downstream with increased entrainment rate, and when the head of entrainment wave catches up with the diesel jet's tip, the whole jet develops into deceleration state, and the distance of jet penetration gradually becomes proportionate to the fourth-root of time. It has been confirmed that the discrete one-dimensional penetration model developed by Musculus and Kattke [13] is capable of predicting the spray penetration after EOI and even for a injection profile with ramping-up and ramping-down processes, and also has been proved to agree well with the experimental data for which the constant injection rate occupied the primary process of injection as well. However, the model is mainly used for diesel spray simulation with quasi-steady state, and also, the one-dimensional discrete spray model cannot intuitively reveal the effects of injection parameters and ambient gas parameters on the spray penetration and the relation between the spray penetration and time variation. In our previous study [14], a zero-dimensional diesel spray model was developed. It is an analytical model for the calculation of spray penetration after EOI, which is derived based on the theoretical analysis of turbulent jet deceleration. However, the spray penetration with time-varying injection rate had not been considered.

For the research of spray propagation with time-varying injection rate, there have also been some attempts. Borée et al. [15] studied the injection velocity profile with a sudden decrease, and a self-similar result was developed based on temporal scaling. However, this model only applies for the cases of a sudden decrease from one constant velocity to another lower velocity. The Desantes packet penetration model[7] was based on the assumption that, once a spray particle is caught up by a speeding subsequently particle, it will travel with a different momentum instantly. Although this approach improved the way for considering varying injection rate, it was not validated by a wide range of injection profiles. Wan et al. [16] proposed a spray penetration model by using scaling parameters. They tested the model with linearly increasing injection rate and showed a good agreement, but they did not think about other injection profiles. Crowe et al. [17] studied the response function of a droplet to the surrounding gas velocity. Their analysis revealed an exponential response function of time for the droplet to reach the surrounding gas velocity. Based on this exponential response function, Abani et al. [18] derived an effective injection velocity instead of actual injection velocity to calculate diesel spray penetration, and the model can be used for various injection profiles. However, the model was only validated by CFD calculation, and it did not consider the spray evolution after EOI.

Thus, the objective of this study is focused on the prediction of the spray propagation after EOI with varying injection rate. For this purpose, the effective injection velocity was introduced into the previous spray model [19] by being defined as the ratio of the momentum flux and fuel mass flow rate integrated over the spray tip cross-sectional area. By this definition, the analysis of effective injection rate and its response time were performed during and after the EOI, so that the fuel mass flow rate and the momentum flux over the spray tip cross-sectional area were derived for varying injection rate even after EOI, thereby the spray penetration was obtained. Finally, the developed model was validated by comparing with experimental data.

2. MODEL DESCRIPTION

2.1 Spray modeling concepts

The research of spray characteristics for varying injection velocity has been investigated for decades. The influences of flow acceleration on turbulent jets were measured by Zhang et al. [20], using injection rate with linear, quadratic and exponential profiles. It was showed that the spray tip evolves with the same form as the forcing function at the nozzle temporally. Breidenthal [21] proposed that for non-steady flow, the exponential function is suitable to be selected as the self-similarity function, and further can be regarded as the response function of a jet particle to the change of nozzle injection rate. Crowe et al. [17] conducted researches on the response function of a droplet in the jet to the ambient gas velocity. They also found that the droplet reaches the ambient gas velocity following an exponential function of time. Based on these researches, Abani et al. [18] developed a relatively simple model, which extended the isolated drop theory in [17] and combined the isolated drop theory with Helmholtz's vortex motion theory [22-24]. Then they indicated that the spray tip responded to the change in injection velocity with an exponential function, which depends on the response time in the spray. As analyzed in [18], the response time of the spray tip τ_V can be obtained as:

$$\tau_v = St\frac{S_{tip}}{u_j} \tag{1}$$

Where St is the Stokes Number, S_{tip} is the spray tip penetration and u_j is the instantaneous injection velocity. When the injection velocity varied from u_{j1} to u_{j2}, the effective injection velocity at the spray tip (u_{eff}) corresponding to the injection velocity change can be calculated as:

$$u_{eff} = u_{j1} + (u_{j2} - u_{j1})\left(1 - \exp\left(\frac{-(t - t_{birth})}{\tau_v}\right)\right) \tag{2}$$

where t_{birth} is the time at when the injection velocity changes, t is the time after the start of injection. In Formula 11, θ is the spray cone angle.

2.2 Concepts modifications

In our previous study [14, 19], the spray tip penetration was derived based on the momentum conservation of gas jet with constant injection rate. As has been demonstrated that, the ratio of momentum flux $\dot{M}_{tip}$ and the mass flow rate $\dot{m}_{f,tip}$ of the fuel integrated over the tip cross-sectional area is important for the prediction of the spray tip penetration. For constant injection rate profile, the value is a constant, and it equals to the fuel injection velocity (u_j) at the nozzle, which can be described as,

$$\frac{\dot{M}_{tip}}{\dot{m}_{f,tip}} = u_j \tag{3}$$

In fact, according to the physical meaning, the ratio of momentum flux and the mass flow rate of the fuel integrated over the tip cross-sectional area can be supposed as a kind of effective injection velocity [18] of the spray tip, which is shown as equation (4).

$$\frac{\dot{M}_{\rm tip}}{\dot{m}_{\rm f,tip}}=u_{\rm eff} \tag{4}$$

During injection, the injection velocity, which is the velocity at the nozzle exit, is the target for effective injection velocity, and the effective injection velocity is varying to the injection velocity with response as an exponential function. Therefore, the effective injection velocity is able to represent the variation of injection velocity on the spray tip. According to the concept, the equation (2) can be re-derived to fit the definition of the effective injection velocity and the calculation of the zero-dimensional spray model [19] as equation (5). For every calculation time step Δt, the effective injection velocity at time $(t+\Delta t)$ is calculated as follows,

$$u_{\rm eff}(t+\Delta t)=u_{\rm eff}(t)+\left(u_{\rm j}(t+\Delta t)-u_{\rm eff}(t)\right)\left(1-\exp\left(\frac{-\Delta t}{\tau_{\rm v}}\right)\right) \tag{5}$$

where $u_{\rm j}(t+\Delta t)$ is the injection velocity at time $(t+\Delta t)$.

After the EOI, the injection is stopped, and the effective injection velocity still varies for a certain time to response to the change of injection velocity. However, the effective injection velocity does not decrease to zero as injection velocity, it will reach a constant velocity finally as the derivation in [14]. Therefore, the target velocity as $u_{\rm j}(t+\Delta t)$ in equation (5) should be replaced by a constant velocity for the situation after EOI ($u_{\rm EOI}$) as follows,

$$u_{\rm eff}(t+\Delta t)=u_{\rm eff}(t)+\left(u_{\rm EOI}-u_{\rm eff}(\rm t)\right)\left(1-\exp\left(\frac{-\Delta t}{\tau_{\rm v}}\right)\right) \tag{6}$$

According to involve the calculation of effective injection velocity demonstrated above into the zero-dimensional spray model [19], a phenomenological spray model with varying injection profile in zero-dimension can be obtained. The spray penetration model developed in present study is based on the previous zero-dimensional spray model [19], thereby the same assumptions are introduced in this model. Firstly, the jet is non-vaporizing and incompressible, meaning that the fuel is universally treated as liquid. Even with this simplification, Naber and Siebers [6] found that non-vaporizing sprays predicted penetration of vaporizing sprays quite well. Secondly, turbulent (and molecular) viscous forces acting on each control volume are neglected, and axial mixing of momentum due to molecular and turbulent diffusion is neglected. Because the normal viscous forces are typically small compared to pressure forces, and viscous shear forces at the outer radius of the jet control volumes can be neglected since velocities and their gradients at the boundary are small. Thirdly, the net force caused by any axial pressure gradient is assumed to be negligible. Moreover, the jet spreading angle is treated as a constant during and after the EOI transient and the shape of the spray is treated to be a cone. Finally, the (normalized) radial profile of mean axial velocity remains unchanged during the EOI transient. Experimental data from at least three separate experiments proved this assumption [25]. Corresponding to this hypothesis, a coefficient β had been derived to account for the effect of radial profiles of the axial velocity and the volume fraction of fuel [13, 25], the value

147 of β varies from 1.0 for a uniform profile to 2.0 for a fully developed spray.

148 **2.3 Calculation before end-of-injection**

149 As has stated above, the spray tip penetration can be divided into two parts by the time of EOI. In order to predict the
150 spray penetration before the EOI, it is of great importance to represent the spray tip velocity with the injection velocity
151 which can be measured in experiments, and the effective injection velocity makes it possible to relate the injection
152 velocity and the spray tip velocity.

153 According to the analysis using vortex theory in [18] and [26], the response time of the eddy at the spray tip to the
154 injection velocity is reasonable to be considered as the characteristic time of the flow in the spray. Therefore, equation (1)
155 for the response time of the spray tip can be rewritten as equation (7).

156 $$\tau_{\mathrm{v}} = St\frac{S_{\mathrm{tip}}(t)}{u_{\mathrm{eff}}(t)} \tag{7}$$

157 As is known to all, the Stokes number (*St*) describes the droplet behavior in a stream. When St is larger than 1.0, the
158 inertial effect plays the major role of the droplet behavior, when St is equal or smaller than 1.0, the diffusion does. In the
159 case of fuel droplets injected from the nozzle, the *St* number is much larger than 1.0 near the nozzle exit because of the
160 high injection pressure, while for the fuel droplets traveled a distance downstream and the diffusion effect is more
161 remarkable, the *St* decreases significantly. Therefore, when the spray is fully-developed, it is logical to treat the *St*
162 number to be 1.0. In this study, the calculation for spray penetration is focused on the period after EOI when the spray is
163 fully developed, so the *St* number is selected to be 1.0.

164 The schematic of the zero-dimensional spray model is shown in Fig. 1. It can be observed that, the spray tip
165 penetration at time $(\mathrm{t}+\Delta\mathrm{t})$ is obtained by,

166 $$S_{\mathrm{tip}}(t+\Delta t) = S_{\mathrm{tip}}(t) + u_{\mathrm{tip}}(t+\Delta t)\cdot\Delta t \tag{8}$$

167 where$u_{\mathrm{tip}}(t+\Delta t)$is the spray tip velocity during the time step from (t) to $(t+\Delta t)$.

168 As for the velocity of the spray tip u_{tip}, it can be obtained as,

169 $$u_{\mathrm{tip}}(t+\Delta t) = \frac{\dot{M}_{\mathrm{tip}}(t+\Delta t)}{\dot{m}_{\mathrm{tip}}(t+\Delta t)} = \beta\bar{u}_{\mathrm{tip}}(t+\Delta t) \tag{9}$$

170 where $\dot{M}_{\mathrm{tip}}(t+\Delta t)$ and $\dot{m}_{\mathrm{tip}}(t+\Delta t)$ are the momentum flux and mass flow rate of injected fluid integrated over the
171 tip cross-sectional area respectively. And $\bar{u}_{\mathrm{tip}}(t+\Delta t)$ represents the average velocity over the tip cross-section.

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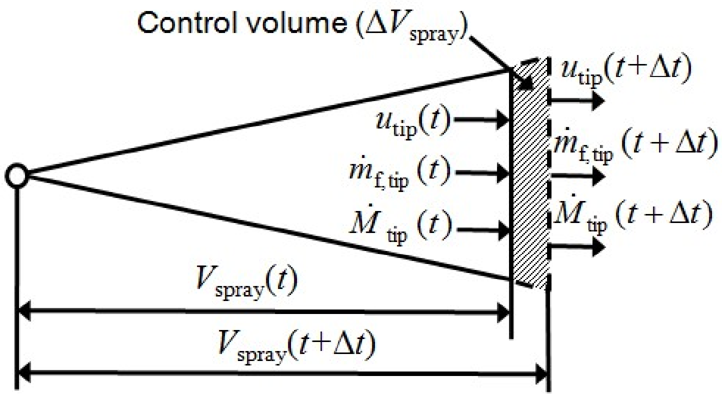

Fig. 1 Schematic of the zero-dimensional spray model

Similar as the analysis in the reference [19], by introducing the factor $\bar{X}_{\text{f,tip}}$, which represents for the average volume fraction of fuel over the cross-section at the spray tip,

$$\bar{X}_{\text{f,tip}}(t+\Delta t) = \frac{\dot{m}_{\text{f,tip}}(t+\Delta t)/\rho_{\text{f}}}{\dot{V}_{\text{spr}}(t+\Delta t)} \tag{10}$$

where ρ_{f} is the liquid fuel density, $\dot{V}_{\text{spr}}(t+\Delta t)$ is the volume change of spray in the time step from (t) to $(t+\Delta t)$, $\dot{V}_{\text{spr}}(t+\Delta t) = V_{\text{spr}}(t+\Delta t) - V_{\text{spr}}(t)$. For the spray at time $(t+\Delta t)$, the volume $V_{\text{spr}}(t+\Delta t)$ is represented as,

$$V_{\text{spr}}(t+\Delta t) = {}^{1}/_{3}\,\pi(\tan({}^{\theta}/_{2}))^2 \cdot (z_0 + S_{\text{tip}}(t+\Delta t))^3 \tag{11}$$

Where z_0 is the distance from the nozzle exit to the virtual spray origin, θ is the spray cone angle.

Substituting equation (9) into equation (4) of the effective injection velocity, $\bar{u}_{\text{tip}}(\text{t}+\Delta\text{t})$ can be derived as,

$$\bar{u}_{\text{tip}}(t+\Delta t) = \frac{\rho_{\text{f}}\bar{X}_{\text{f,tip}}(t+\Delta t)u_{\text{eff}}(t+\Delta t)}{\rho_{\text{tip}}(t+\Delta t)} \tag{13}$$

So far the velocity of the spray tip $u_{\text{tip}}(t+\Delta t)$ can be obtained based on the equations (9) to (13),

$$u_{\text{tip}}(t+\Delta t) = \rho_{\text{f}}\bar{X}_{\text{f,tip}}(t+\Delta t)u_{\text{eff}}(t+\Delta t)\beta/\rho_{\text{tip}}(t+\Delta t) \tag{14}$$

So that the spray penetration before transition time is able to be obtained by substituting equation (14) into equation (8).

2.4 Calculation after the end-of-injection

In order to conduct the calculation of spray penetration before and after the EOI, it is an essential precondition to determine the timing of the EOI. In this study, the end of injection is defined by introducing a parameter R_{IV} that represents the influence of the injection rate on spray penetration. For example, in the case of a triangle injection profile, the injection velocity decreases after achieving a peak. When the injection velocity is too low to affect the spray tip penetration, it can be treated as the termination of injection.

In fact, the factor of R_{IV} is proposed in accordance to the definition of the time constant, or response time. Because the response time represents the phenomenon when the system reaches 63.2% of its final (asymptotic) value, the remaining influence can be ignored. While R_{IV} describes the decay extent when the injection velocity is too low to affect the spray tip penetration, which is similar with the physical meaning of response time.

Therefore, in this study, a dimensionless ratio (R_{IV}) between the time ratio t_{de}/t_{decay} and the time ratio t_{decay}/t_{inj} is proposed to define the injection velocity (u_{EOI}) at the end of injection, where t_{de} is the time from start of decay to the defined injection termination, t_{decay} is the period that injection velocity decays, and t_{inj} is the injection duration, as shown in Fig. 2.

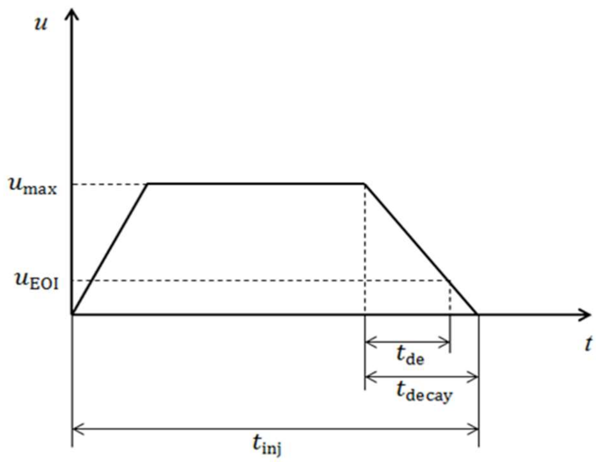

Fig. 2 Schematic of maximum injection velocity and injection velocity at the end of injection

It can be seen from Fig. 2, the time ratio t_{de}/t_{decay} represents the decay extent of the injection velocity, and the time ratio t_{decay}/t_{inj} stands for the proportion of the decay period in the injection duration. In this study, it is supposed that when the ratio of t_{de}/t_{decay} and t_{decay}/t_{inj} reaches 0.632, which is determined in accordance to the value of response time, the injection velocity becomes too low to affect the spray tip penetration.

Therefore, the analytical equation of injection velocity at the EOI can be derived as follows,

$$u_{EOI} = (1 - R_{IV} \times t_{decay}/t_{inj})\, u_{max} \quad (14)$$

whereR_{IV} is selected to be 0.632, and this value is applied in the following model.

In this way, the time, when the injection velocity decreases to u_{EOI}, is defined as the end-of-injection (t_{EOI}). After the end of injection, there is a turning point named transition time, which is proposed by Musculus and Kattke [25] and is defined as the time when entrainment wave arrives at the spray tip and the whole diesel jet goes into deceleration state. The transition time has been observed to be twice of the injection duration in reference [25], which is based on experimental data [28, 29] from steady jets. Nevertheless, it has not been studied for the case of varying injection rate. In this study, to simplify the calculation, the transition time is still treated as twice of the injection duration, which is calculated by the new defined time of the end-of-injection.

After the EOI, the u_{EOI} in equation (6) is calculated by equation (14), then the effective injection velocity can be obtained. When the time gets equal to or larger than transition time, the effective injection velocity $u_{\mathrm{eff}}(t)$ becomes u_{EOI}, and the right side of the equation becomes $u_{\mathrm{eff}}(t)$. The equation can be simplified to,

$$\mathrm{u}_{\mathrm{eff}}(t+\Delta t)=\mathrm{u}_{\mathrm{eff}}(t) \tag{15}$$

Before transition time, the spray tip penetration is calculated using the equation (6), (14) and equation (8) to (13). After the transition time, same with the analysis of reference [19], the same format equation (12) for the average velocity over the tip cross-section is also available in this study, nevertheless, $\bar{X}_{\mathrm{f,tip}}(t+\Delta t)$ and $\rho_{\mathrm{tip}}(t+\Delta t)$ are replaced by $\bar{X}_{\mathrm{f,tip,atr}}(t+\Delta t)$ and $\rho_{\mathrm{tip,atr}}(t+\Delta t)$ respectively after the transition time, which are given by,

$$u_{\mathrm{tip}}(t+\Delta t)=\beta\frac{\rho_{\mathrm{f}}\bar{X}_{\mathrm{f,tip,atr}}(t+\Delta t)u_{\mathrm{eff}}(t+\Delta t)}{\rho_{\mathrm{tip,atr}}(t+\Delta t)} \tag{16}$$

Where the subscript ‘atr’ means ‘after the transition time’.

Because the information of end of injection has reached the spray tip, some entrained air flows into fuel region and mixes with fuel, and then the mixture in the fuel and air region flows out through the tip. Owing to the fluid continuity, the volumetric flow rate of the mixture flows out is equal to that of the air flows in. So that the fuel volume fraction flowing out of the fuel and air region from the tip equals to the average volume fraction X_{fcm} of fuel in the fuel and air region, which can be calculated,

$$X_{\mathrm{fcm}}=\frac{m_{\mathrm{f}}/\rho_{\mathrm{f}}}{m_{\mathrm{f}}/\rho_{\mathrm{f}}+\beta u_{\mathrm{eff}}(t+\Delta t)A_0\cdot(t-2t_{\mathrm{EOI}})} \tag{17}$$

where m_{f} is the total injected fuel mass, t_{EOI} is the time of end of injection, and $2t_{\mathrm{EOI}}$ is quite the transition time.

Finally, $\bar{X}_{\mathrm{f,tip,atr}}(t+\Delta t)$ is derived as,

$$\bar{X}_{\mathrm{f,tip,atr}}(t+\Delta t)=\beta\frac{X_{\mathrm{fcm}}u_{\mathrm{eff}}(t+\Delta t)A_{0}\cdot\Delta t}{V_{\mathrm{spr}}(t+\Delta t)-V_{\mathrm{spr}}(t)} \tag{18}$$

So that the spray penetration after the transition time is able to be calculated using equation (8).

3. RESULTS AND DISCUSSION

3.1 Validation for quasi-steady spray with ramping-up and ramping-down injection rate

In the previous spray models that mentioned above [13, 14, 19, 25], the injection rate is assumed to be a constant value and remains unchanged during the injection. However, in real diesel engines, most injection strategies are quasi-steady with ramping-up and ramping-down injection rate. The ramping-up and ramping down process takes up a small part, approximately 10% of the whole injection duration, but it does make a difference in the spray penetration.

In order to validate the feasibility of the spray model developed, the calculation is conducted and compared with the experimental data. It needs to be noted here, the experimental data are taken from the engine combustion network (ECN), and the experimental conditions are listed in Table 1. In particular, the value of R_{IV} is chosen similar with the response time to be 0.65, and the value of Stokes number is selected to be 1.0, as has mentioned above.

As for the selection of the value of parameters β, the detail explanation is given as follows.

Based on the assumption stated above, the mean axial velocity is regarded to be unchanged during and after the EOI transient. In real jets, the mean velocity and volume fraction is non-uniform over the cross- sectional area, so a factor β is needed to account for the shape of the fuel volume fraction and velocity profiles. According to the analysis of β in the reference [25], the value of β is varied from 1.0 for a uniform profile to approximately 2.0 for a fully developed jet. In this study, the main attention is paid on the penetration of a fully-developed spray. Therefore, for the simplicity of calculation, the value of β is set to be a constant 2.0.

Figure 3 shows the comparison of experimental data and calculated results including varying and constant injection rates. When the injection rate is considered as a constant (rectangular injection profile) for calculation, the injection duration is from start of injection to start of decay. It can be found that the calculation with varying injection rate capture the same spray-penetrating tendency and almost the same penetration value with the experimental data, however, the calculation with constant injection rate shows obvious larger spray tip penetration than that of experiment, and the difference increases after EOI. That is caused by the higher effective injection velocity in the case of constant injection rate. According to the reference [14], in the case of constant injection rate, the effective injection velocity is equal to that during injection and the value is same with injection velocity whatever before and after EOI. As shown in Fig. 4, the injection rate of the rectangle profile is higher than the maximum injection rate of the trapezoid profile and the average level of the experimental injection rate, that reals the same situation of injection velocity. Therefore, the effective

264 injection velocity with rectangle injection profile is higher than that with trapezoid injection profile. Moreover, the effective injection velocity after EOI is smaller than the maximum injection velocity in the case with ramping-down injection rate based on equation (14), so that the effective injection velocity after EOI is also smaller than that of constant injection rate.

268 The comparison indicates that the zero-dimensional spray model developed in this study is capable of predicting the spray penetration of quasi-steady injection with ramping-up and ramping-down process. On the other hand, even for the quasi-steady spray, the ramping-up and ramping-down process has an important influence on the spray penetration and should not be ignored, otherwise the spray tip penetration could be overestimated.

Table 1 Initial conditions of calculation

Injection quantity	3.46mg
Injection duration	1.54ms
Ambient gas temperature	440.0 K
Ambient gas pressure	2.93 MPa
Ambient gas composition by volume	N_2: 100%
Nozzle diameter	0.09mm
Orifice flow coefficient	0.98
Stokes number	1.0
R_{IV}	0.65
β	2.0
Spreading angle	22˚

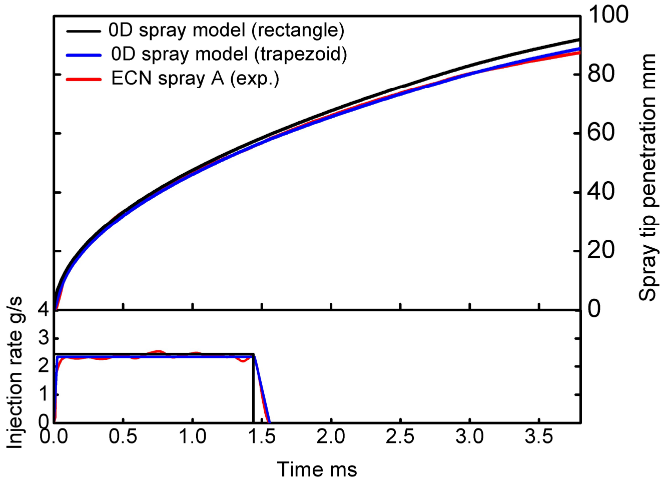

Fig. 3 Spray tip penetration
(Injection quantity: 3.46 mg)

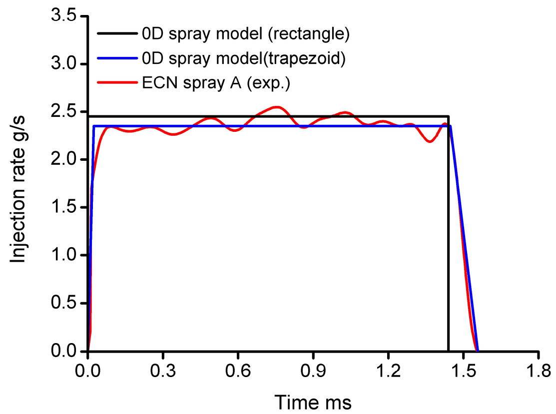

Fig. 4 Injection rates

3.2 Validation for transient spray with triangle injection profile

Because of the entrainment wave effects proposed by Musculus and Kattke [13], shortening injection duration can enhance the fuel-air mixing and decrease the spray penetration, which is useful for improving the diesel spray combustion and wet-impingement phenomenon in advanced combustion modes. Therefore, an effective approach to shorten the injection duration is to elevate the injection pressure, however, the ramping-up and ramping-down processes are dominant during injection when the injection quantity is small. In the case of large injection amount, dividing the fuel injection into several short injection stages is a good choice for improving the diesel spray combustion, such as the multiple-injection strategy. For multiple-injection strategy, the pilot and post injection has very short injection duration, so that the injection quantity is very small and it causes the ramping-up and ramping-down processes take up the primary process of injection. As shown in Fig. 4, the injection rates of different injection quantities were measured by Zeuch injection rate meter, and the back pressure for injection rate measurement was 4 MPa. The injection system is a common-rail injection system, and the injector has 6 nozzle holes with the nozzle diameter of 0.18 mm. The injection pressures (P_{inj}) are 130 MPa and 180 MPa. In the cases of small injection quantity with short injection duration, such as 2.5 mg and 5.0 mg, the injection rate only has ramping-up and ramping-down processes, which can be treated as triangle shape. It makes the spray stay in the transient state during injection.

To prove the model valid for predicting the spray penetration with transient state, such as triangle injection rate, the experiments of non-combustion and non-evaporation diesel spray are performed on a constant-volume combustion vessel with short injection duration, and the spray penetrations are measured based on shadowgraph image. The experimental system of the combustion vessel is same with that in the reference [30]. The combustion vessel is cylindrical, and the volume is 150 cm^3 with diameter of 80 mm and depth of 30 mm. Diesel fuel is injected into the chamber using a common-rail injection system via an injector with orifice diameter of 0.18 mm with single nozzle hole. The operating conditions are listed in Table 2, and calculations are performed based on these conditions. The ambient gas is Nitrogen, and the ambient pressure and temperature is 1.0 MPa and 293K respectively. The fuel is diesel fuel of JIS 2#. Fuel injection pressure is 130MPa, and the injection quantity of 0.928 mg is conducted for spray experiment. The injection quantity corresponds to the fuel amount of 5.6mg injected by the injector with six nozzle holes. However, the actual injection duration is 0.103 ms, it is much shorter than that of 5.0 mg shown in Fig. 5. This is because that the injection rate of the injector with single nozzle hole is higher than the injector with multi-hole.

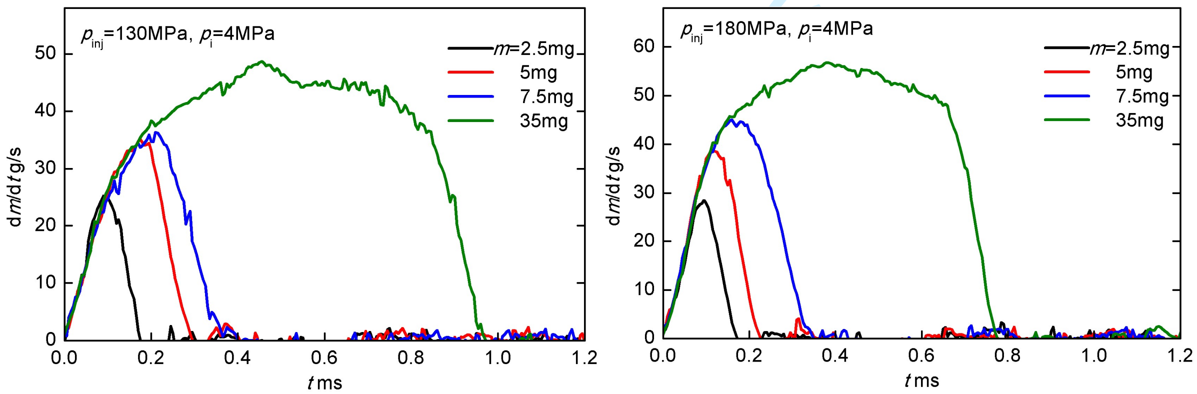

Fig.5 Injection ratemeasured by Zeuch method.

308 Table 2 Experimental conditions

Items	Parameters
Ambient gas temperature	293K
Ambient gas pressure	1.0 MPa
Ambient gas composition by volume	N_2: 100%
Fuel	JIS 2#, Cetane Index: 55
Injection Pressure	130 MPa
Injection Quantity	0.928 mg
Nozzle diameter	0.18
Orifice flow coefficient	0.8
Stokes number	1.0
R_{IV}	0.65
β	2.0

309

310 The spray behavior was imaged using shadowgraph, and the configuration of shadowgraph imaging system is shown
311 in Fig. 6. The light source is the Xeon lamp, and the two concave mirrors have the same focal length that is 1910 mm.
312 The shooting speed of the high-speed camera is 50000 fps, and the time of exposure is 5 us. The images of the sprays
313 with injection quantity of 0.928 mg are shown in Fig. 7. The spreading angle of sprays is 12°, and the calculated spray
314 penetrations and the comparison with measured spray penetrations are revealed in Fig. 8.

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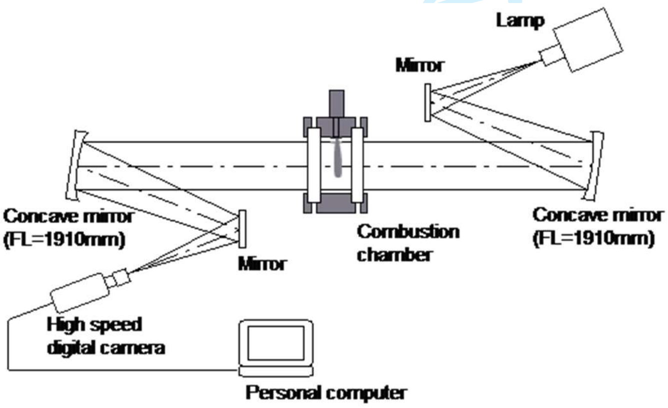

Fig.6 Schematic of shadowgraph imaging system.

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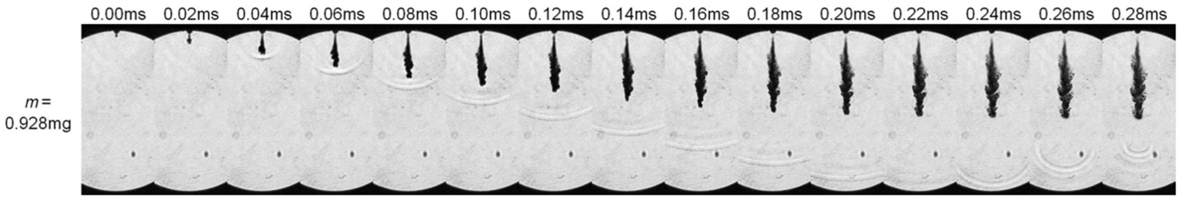

Fig.7 Shadowgraph images of sprays with short injection durations.

As shown in Fig.8, the upper graphs are the spray tip penetrations and the lower graphs are the injection rates. There are two profiles of injection rate considered for calculation, triangle and rectangle. The triangle injection rate is to mimic the measured injection rate, the slopes of ramping-up and ramping-down processes for calculation are the average slopes of those in real injection rate. The rectangle injection rate is used to involve case of constant injection rate for comparison. The timing of EOI for rectangle injection rate is that of start of injection decay. It can be observed that much higher injection velocity for the rectangle injection rate due to short injection duration, consequently the spray tip penetration calculated with rectangle injection rate is much larger than that calculated with triangle injection rate and measured by experiment. However, the spray tip penetration calculated with triangle injection rate shows a good agreement with experimental data. The results of comparison reveal that the ramping-up and ramping-down injection processes affect transient spray propagation greatly, it is necessary to involve the effect of varying injection rate for accurate estimation of spray tip penetration, which is also helpful to promote the accuracy of entrainment and mixing calculations.

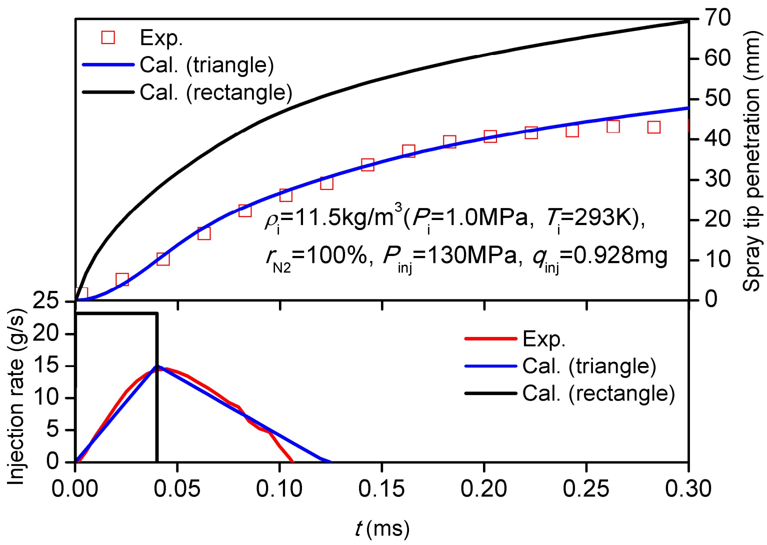

Fig.8 Spray tip penetration (q_{inj}=0.928 mg).

For further validation of the improved spray model with various injection conditions and ambient gas conditions, also for the validation of the rationality of parameter selection, especially *St*, R_{IV} and β, the experiments were conducted using the same experiment setup as before, but with different injection and ambient gas conditions. The ambient gas composition was changed, the ambient temperature was increased greatly to 1015K, and the ambient pressure also increased to 4.0MPa. The injection pressure was 130MPa and 180MPa and injection qualities was 0.43mg and 0.83mg respectively. Detailed information for the experiments and calculation conditions are listed in Table 3.

Table 3 Calculation conditions

Items	Parameters
Ambient gas temperature	1015K
Ambient gas pressure	4.0 MPa
Ambient gas composition by volume	O_2: 0.5%, N_2: 87.4%, CO_2: 4.8%, H_2O: 7.3%
Fuel	JIS 2#, Cetane Index: 55
Injection Pressure	180 MPa
Injection Quantity	0.43 mg, 0.83mg
Nozzle diameter	0.18
Orifice flow coefficient	0.8
Stokes number	1.0
R_{IV}	0.65
β	2.0

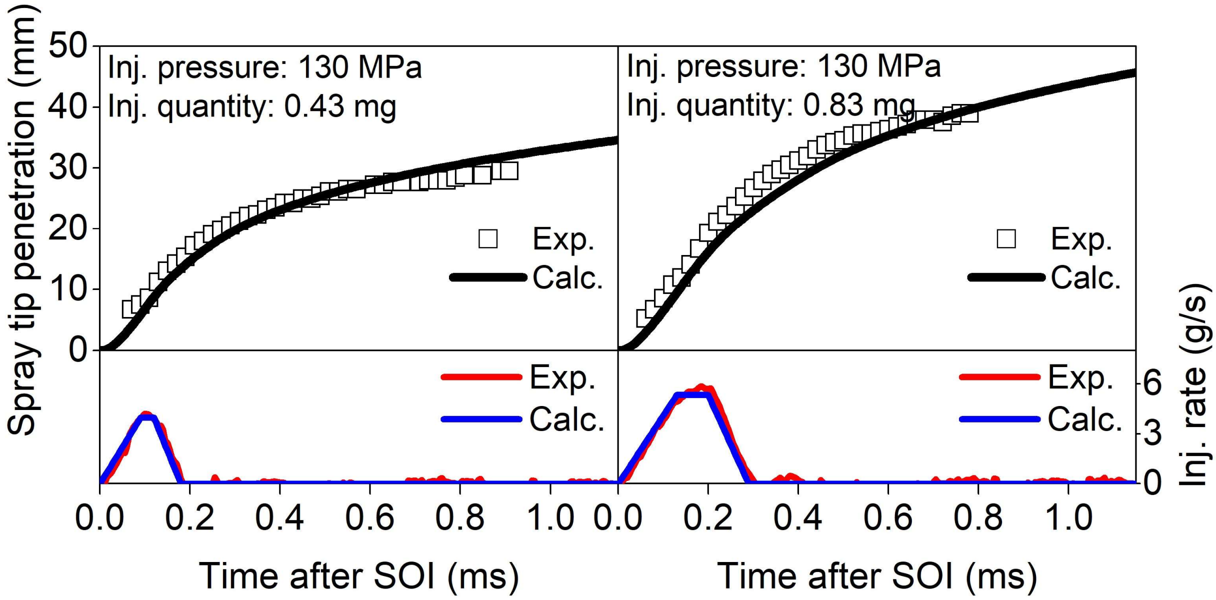

Fig.9 Spray tip penetration (P_{inj}=130MPa).

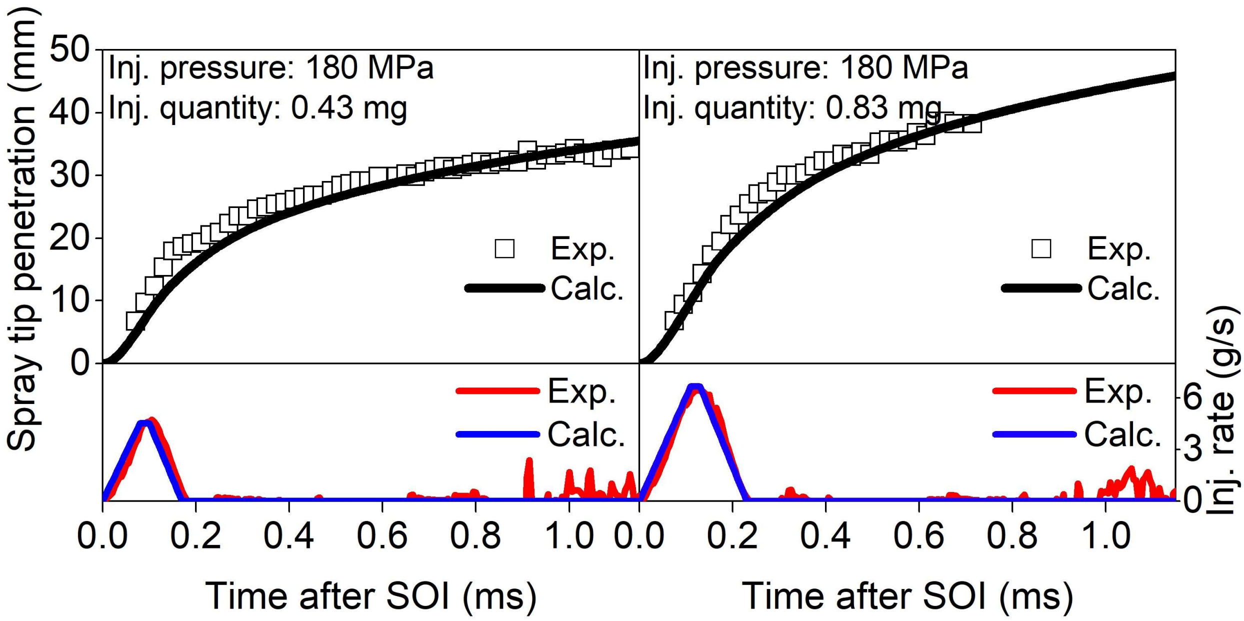

Fig.10 Spray tip penetration (P_{inj}=180MPa).

Fig.9 shows the comparison between experimental and calculated spray penetrations and injection rates for injection pressure of 130 MPa, and Fig. 10 is for 180MPa. As the shown in these two figures, the input injection rates are similar with the measured injection rates, and the calculated penetrations obtains good agreements with experimental spray penetrations. The model constants including *St*, R_{IV} and β have not been changed to adjust the calculation to fit the experimental results. The reason is that the constants in the developed model are defined based on spray and turbulent jet theories, they are selected correctly and reasonable to be adopt to calculate diesel spray penetration under diesel engine-like conditions.

As a result, it is reasonable to conclude that the developed zero-dimensional spray model is able to predict the whole spray tip penetration for the transient spray. The experimental data is not rich for spray model validation. However, the spray model proposed in this study is derived based on the momentum theory that has been applied for spray modelling and validated. Thus, the developed spray model is valid for various injection conditions for diesel engines.

4. CONCLUSIONS

In this study, a zero-dimensional spray model was developed to evaluate the spray tip penetration after the end of injection for time-varying injection profile. By introducing the definition of effective injection velocity into a developed zero-dimensional spray model and combined with the vortex theory, the effective injection velocity and its response time to the injection velocity were derived. After that, the target injection velocity after EOI and the timing of EOI were defined and derived. Thereby the spray penetration with varying injection profile even after EOI can be obtained based on the momentum theory. The model was tested with the experimental data of steady and unsteady sprays, and the results showed to fit the data very well.

The primary conclusions are summarized as follows:

1. The effective injection velocity is introduced as the ratio between momentum flux and fuel mass flow over tip cross-sectional area of spray. It is proved to be reasonable and workable for the calculation of spray tip penetration with time-varying injection profile.
2. The effective injection velocity after EOI gradually changes to a constant. The constant is defined as the target injection velocity after the EOI, and it is derived as $u_{\mathrm{EOI}} = (1 - R_{\mathrm{IV}} \times t_{\mathrm{decay}}/t_{\mathrm{inj}})\, u_{\mathrm{max}}$. The EOI timing is occurred at when the injection velocity decreases to the value of target injection velocity.
3. The developed zero-dimensional diesel spray model has capability to accurately capture the evolution of diesel spray penetration with time-varying injection profiles as shown by comparing with the experimental data of steady and unsteady sprays.

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